

C 22091

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Name.....

Reg. No.....

## SECOND SEMESTER (CBCSS-UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 2B 02—CALCULUS OF SINGLE VARIABLE—I

(2021 Admissions)

Time : Two Hours and a Half

Maximum Marks : 80

## Section A

*Answer at least ten questions.**Each question carries 3 marks.**All questions can be attended.**Overall Ceiling 30.*

1. Let  $f(x) = \sin x$  and  $g(x) = 1 - 2x$ . Find the functions  $g \circ f$  and  $f \circ g$ . What are their domains ?
2. Find  $\lim_{x \rightarrow -3} \frac{x^2 + 2x - 3}{x^2 + 4x + 3}$ .
3. Let  $H(t) = \begin{cases} 0 & \text{if } t < 0 \\ 1 & \text{if } t \geq 0. \end{cases}$  Determine whether H is continuous from the right at 0 and/or from the left at 0.
4. Find  $\lim_{x \rightarrow 0} \frac{\sin 2x}{3x}$ .
5. Find the instantaneous rate of change of  $f(x) = \frac{2}{x} + x$  at  $x = 1$ .
6. Find the derivative of  $f(x) = 3\sqrt{x} + 2e^x$ .
7. Find the critical points of  $f(x) = x - 3x^{1/3}$ .
8. State Mean value theorem.
9. Find  $\lim_{x \rightarrow -\infty} \frac{x^2 + 1}{x - 2}$ .
10. Find the horizontal and vertical asymptotes of  $f(x) = \frac{1}{x + 2}$ .
11. Find  $\int \frac{2x^2 - 1}{x^2} dx$ .

Turn over

12. Find  $\int \frac{e^{2/x}}{x^2} dx$ .
13. Evaluate  $\int_{-1}^2 |x| dx$ .
14. Find the area of the region between the graphs of  $y = e^x$  and  $y = x$  and the vertical lines  $x = 0$  and  $x = 1$ .
15. Find the work done by the force  $F(x) = 3x^2 + x$  in moving a particle along the  $x$ -axis from  $x = 2$  to  $x = 4$ .

(10 × 3 = 30 marks)

### Section B

*Answer at least **five** questions.  
Each question carries 6 marks.  
All questions can be attended.  
Overall Ceiling 30.*

16. Find  $\lim_{x \rightarrow 0} x^2 \sin \frac{1}{x}$ .
17. Let  $f(x) = 2x^3 + x$  (a) Find  $f'(x)$ . (b) What is the slope of the tangent line to the graph of  $f$  at  $(2, 18)$ ; (c) How fast is  $f$  changing when  $x = 2$ .
18. Find the relative extrema of  $f(x) = x^3 - 3x^2 - 24x + 32$  using second derivative test.
19. Let  $f(x) = x^3 - x$  for  $x$  in  $[-1, 1]$  :
- (a) Show that  $f$  satisfies the hypothesis of Rolle's theorem on  $[-1, 1]$ .
- (b) Find the numbers  $c$  in  $(-1, 1)$  such that  $f'(c) = 0$  by Roll's theorem.
20. (a) In a test run of a maglev along a straight elevated monorail track, data obtained from reading its speedometer indicated that the velocity of the maglev at time  $t$  can be described by the velocity function  $v(t) = 8t, 0 \leq t \leq 30$ . Find the position of the maglev. Assume that the maglev is initially located at the origin of a co-ordinate line.
- (b) Find  $\int \frac{dx}{1 - \sin x}$ .
21. (a) State fundamental theorem of Calculus.
- (b) Find  $\frac{d}{dx} \left[ \int_1^x t^3 dt \right]$  by using the above theorem and by performing the integration and differentiation.

22. Let  $R$  be the region bounded by the graphs of  $x = -y^2 + 6y$  and  $x = 0$ . Find the volume of the solid obtained by revolving  $R$  about the  $x$ -axis.
23. Find the area of the surface obtained by revolving the graph of  $x = y^3$  on the interval  $[0, 1]$  about  $y$ -axis.

(5 × 6 = 30 marks)

### Section C

Answer any **two** questions.  
Each question carries 10 marks.

24. (a) By translating the graph of  $y = x^2$ , sketch the graphs of  $y = x^2 + 2$  and  $y = (x - 2)^2$ .

(b) Let  $f(x) = \begin{cases} -x^5 + x^3 + x + 1 & \text{if } x < 0 \\ 2 & \text{if } x = 0 \\ x^2 + \sqrt{x+1} & \text{if } x > 0 \end{cases}$

Find  $\lim_{x \rightarrow 0^+} f(x)$  and  $\lim_{x \rightarrow 0^-} f(x)$ . Does  $\lim_{x \rightarrow 0} f(x)$  exist. Justify your answer.

25. Sketch the graph of the function  $f(x) = 2x^3 - 3x^2 - 12x + 12$ .

26. Using the definition of the definite integral evaluate  $\int_{-1}^3 (4 - x^2) dx$ .

27. (a) Find the area of the region enclosed by the graphs of  $y = \frac{x^2}{4}$  and  $y = \frac{8}{x^2 + 4}$ .

- (b) Find the volume of a right pyramid with a square base of side  $b$  and height  $h$ .

(2 × 10 = 20 marks)