

C 4162

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Name.....

Reg. No.....

**SECOND SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION  
APRIL 2021**

Mathematics

MAT 2C 02—MATHEMATICS

Time : Three Hours

Maximum : 80 Marks

**Part A (Objective Type Questions)***Answer all twelve questions.**Each question carries 1 mark.*

1. Find  $\int a^x dx$ .
2. Define the partial derivative of  $f(x, y)$  with respect to  $x$  at  $(x_0, y_0)$ .
3. If  $f$  is continuous on  $[a, \infty)$ , then  $\lim_{b \rightarrow \infty} \int_a^b f(x) dx = \dots\dots\dots$
4. If a series  $\sum a_n$  converges, then  $\lim a_n = \dots\dots\dots$
5. Find the  $n^{\text{th}}$  term of the sequence  $-2, 2, -2, 2, \dots\dots$
6. Find the domain of the function  $z = \sin xy$ .
7. The polar form of the line  $y = 2$  is  $\dots\dots\dots$
8.  $\frac{d}{dx} \cosh x = \dots\dots\dots$
9. If  $f(x, y) = 2x^2y$  then find  $\frac{\partial^2 f}{\partial x \partial y}$ .
10. Give an example of conditionally converging series.

**Turn over**

11. State Sandwich theorem for sequences.
12. Write the transformation equations for Cartesian co-ordinates to spherical polar co-ordinates.

(12 × 1 = 12 marks)

**Part B (Short Answer Type)**

*Answer any **nine** questions.  
Each question carries 2 marks.*

13. Evaluate  $\int_0^{\log 2} 4e^x \sinh x \, dx$ .
14. The region between the curve  $y = \sqrt{x}$ ,  $0 \leq x \leq 4$  and the  $x$ -axis is revolved about the  $x$ -axis to generate a solid. Find its volume.
15. Determine whether the sequence  $a_n = \frac{2n+1}{3n+1}$  is non-decreasing and if it is bounded from above.
16. Describe the level surface of the function  $f(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 1}$ .
17. The plane  $x = 1$  intersects the paraboloid  $z = x^2 + y^2$  in a parabola. Find the slope of the tangent to the parabola at  $(1, 2, 5)$ .
18. Find  $\frac{\partial z}{\partial y}$  if the equation  $yz - \ln z = x + y$  defines  $z$  as a function of the two independent variables  $x$  and  $y$  and the partial derivatives exists.
19. Is the area under the curve  $y = \ln x/x^2$  from  $x = 1$  to  $x = \infty$  finite? If so, what is it?
20. Evaluate  $\int_0^1 \frac{dx}{\sqrt{3+4x^2}}$ .
21. Write the polar form of the circle  $x^2 + (y-3)^2 = 9$ .
22. Draw the polar curve  $r = 2 \cos \theta$ .

23. Find a spherical co-ordinate equation for the sphere  $x^2 + y^2 + (z - 1)^2 = 1$ .

24. Find the volume of  $\sum_{n=1}^{\infty} \frac{1}{n(n+1)}$ .

(9 × 2 = 18 marks)

**Part C (Short Essay Type)**

Answer any **six** questions.

Each question carries 5 marks.

25. Find the volume of the solid generated by revolving the region bounded by  $y = \sqrt{x}$  and the lines  $y = 1, x = 4$  about the line  $y = 1$ .

26. Investigate the convergence of  $\int_0^3 \frac{dx}{(x-1)^{2/3}}$ .

27. Does the sequence whose  $n$ th term is  $\left(\frac{n+1}{n-1}\right)^n$  converge? If so, find its limit.

28. Find the sum of the series  $\sum_{n=1}^{\infty} \frac{2^{n-1} - 1}{3^n}$ .

29. Find the linearization of  $f(x, y, z) = x^2 - xy + 3 \sin z$  at the point  $(2, 1, 0)$ .

30. Express  $\frac{\partial w}{\partial r}$  and  $\frac{\partial w}{\partial s}$  in terms of  $r$  and  $s$  if  $w = 2x + 2y - z^2, x = r/s, y = r^2 + \ln s, z = 2r$ .

31. Find the length of the curve  $x = \frac{y^3}{3} + \frac{1}{4y}$  from  $y = 1$  to  $y = 3$ .

**Turn over**

32. Show that  $\sum_{n=1}^{\infty} \frac{\ln n}{n^{3/2}}$  converges.

33. Find the radius and interval of convergence of the series  $\sum_{n=0}^{\infty} (-1)^n (4x+1)^n$ .

(6 × 5 = 30 marks)

### Part D (Essay Type)

Answer any **two** questions.

Each question carries 10 marks.

34. Find the length of the cardioid  $r = 1 - \cos \theta$ .

35. a) Evaluate  $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ .

b) Show that  $\tanh^2 x = 1 - \operatorname{sech}^2 x$

36. a) Using partial differentiation find  $w'(0)$  if  $w = xy + z$ ,  $x = \cos t$ ,  $y = \sin t$ ,  $z = t$ .

b) If  $f(x - y, y - z, z - x) = 0$ , show that  $\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z} = 0$ .

(2 × 10 = 20 marks)