C 4388 **(Pages : 3)** **Name.......................................**

Reg. No...................................

SECOND SEMESTER (CBCSS—UG) DEGREE EXAMINATION APRIL 2021

Mathematics

MAT 2C 02—MATHEMATICS—2

(2019 Admissions)

Time : Two Hours Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 marks.
All questions can be attended.
Overall Ceiling 24.

1. Compute the derivative of $\sqrt{x}$ using inverse function rule. Evaluate the derivative at $x = 2$.

2. Convert the relation $r = 1 + 2\cos\theta$ to Cartesian co-ordinates.

3. Compute $\int \cosh^2 x \, dx$.

4. Find $\frac{d}{dx} \cosh^{-1} \sqrt{x^2 + 1}, x \neq 0$.

5. Find $\int_{-\infty}^{\infty} \frac{dx}{1 + x^2}$.

6. Show that $\int_0^{\infty} \frac{\sin x}{\left(1 + x^2\right)} dx$ converges.

7. A bouncing ball loses half of its energy on each bounce. The height reached on each bounce is proportional to the energy. Suppose that the ball is dropped vertically from a height of one meter. How far does it travel ?

Turn over

8. State Ratio comparison test and show that $\sum_{i=1}^{\infty} \frac{2}{4+i}$ diverges.

9. Prove that the vectors $w_1 = \left(\frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}\right), w_2 = \left(\frac{-2}{\sqrt{6}}, \frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$ and $w_3 = \left(0, \frac{1}{\sqrt{2}}, \frac{-1}{\sqrt{2}}\right)$ are orthonormal vectors.

10. Define basis of a vector space. Give a basis for vector space P_n of all polynomial of degree less than or equal to n.

11. Find the inverse of $A = \begin{pmatrix} 1 & 8 \\ 2 & 10 \end{pmatrix}$.

12. Find the eigenvalues and eigenvectors of $A = \begin{pmatrix} 3 & 4 \\ -1 & 7 \end{pmatrix}$.

(8 × 3 = 24 marks)

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Let $f(x) = x^2 + 2x + 3$. Restrict f to a suitable interval so that it has an inverse. Find the inverse function and sketch its graph.

14. Find the length of the graph of $f(x) = (x-1)^{3/2} + 2$ on $[0, 2]$.

15. State root test and test the convergence for the series $\sum_{n=1}^{\infty} \frac{2^n}{n^3}$.

16. For which x does the series $\sum_{n=0}^{\infty} \frac{4^n}{\sqrt{2n+5}} (x+5)^n$ converge.

17. Let $u_1 = (1, 1, 1)$, $u_2 = (1, 2, 2)$ and $u_3 = (1, 1, 0)$ be basis of $\mathbb{R}^3$. Using Gram Schimdt process find an orthonormal basis of $\mathbb{R}^3$.

18. Compute A^m for $A = \begin{pmatrix} 1 & 1 & -2 \\ -1 & 2 & 1 \\ 0 & 1 & -1 \end{pmatrix}$.

19. Identify the conic whose equation is $2x^2 + 4xy - y^2 = 1$.

(5 × 5 = 25 marks)

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. (a) Find the area of the surface obtained by revolving the graph $y = x^2$ about the y-axis for $1 \le x \le 2$.

    (b) Determine whether the set of vectors $u_1 = (1, 2, 3)$, $u_2 = (1, 0, 1)$ and $u_3 = (1, -1, 5)$ is linearly dependent or linearly independent.

21. (a) Find the terms through cubic order in the Taylor series for $\frac{1}{1+x^2}$ at $x_0 = 1$.

    (b) Find an LU factorization of $A = \begin{pmatrix} -1 & 2 & -4 \\ 2 & -5 & 10 \\ 3 & 1 & 6 \end{pmatrix}$.

(1 × 11 = 11 marks)