C 21546 (Pages : 2) Name..

Reg. No.....................................

FOURTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4C 04—MATHEMATICS – 4

(2019 Admission onwards)

Time : Two Hours   Maximum : 60 Marks

Section A

Answer at least **eight** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 24.

1. Write the order and degree of the differential equation $\dfrac{d^2 y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 + 4y = \sin x$.

2. Verify that $y = xe^x$ is a solution of $y'' - 2y' + y = 0$.

3. Show that $\left(25x^2 - 5y\right)dx + \left(3y^2 - 5x\right)dy = 0$ is an exact differential equation.

4. Find the integrating factor corresponding to the differential equation $\dfrac{dy}{dx} + y\tan x = \cos x$.

5. Reduce $\dfrac{dy}{dx} = \left(y - 2x^2\right) - 7$ to an equation with separable variables.

6. Find the general solution of $y'' - y' - 2y = 0$.

7. Find the particular integral of $y'' + 5y' + 6y = e^{2x}$.

8. Find the Laplace transform of $\sin 3t \cos 2t$.

9. Find the Laplace transform of $e^{-3t} t^3$.

10. Write the inverse Laplace transform of $\dfrac{s}{s^2 + 16}$.

11. Show that the functions $f_1(x) = x^3$ and $f_2(x) = x^2 + 1$ are orthogonal on $[-1, 1]$.

12. Show that the partial differential equation $3\dfrac{\partial^2 u}{\partial x^2} = \dfrac{\partial u}{\partial y}$ is parabolic.

(8 × 3= 24 marks)

Turn over

Section B

Answer at least **five** *questions.*
Each question carries 5 *marks.*
All questions can be attended.
Overall Ceiling 25.

13. Solve $(1+x)y\,dx + (1-y)x\,dy = 0$.

14. Solve $(x^2 + y^2)\dfrac{dy}{dx} = xy$.

15. Solve $y'' + y = \tan x$ using the method of variation of parameter.

16. Find the Laplace transform of $\dfrac{1-\cos t}{t^2}$.

17. Find the inverse Laplace transform of $\dfrac{s^2 + 2s + 5}{s^3}$.

18. Apply convolution theorem to evaluate the inverse Laplace transform of $\dfrac{s^2}{\left(s^2 + a^2\right)\left(s^2 + b^2\right)}$.

19. Solve $\dfrac{\partial u}{\partial x} - 2\dfrac{\partial u}{\partial y} - u = 0$ using method of separation of variables.

$(5 \times 5 = 25 \text{ marks})$

Section C

Answer any **one** *question.*
The question carries 11 *marks.*

20. Solve $x^3 y''' - x^2 y'' + 2xy' - 2y = \cos(2\log x)$.

21. Expand $f(x) = x\sin x$ as a Fourier series in $0 < x < 2\pi$.

$(1 \times 11 = 11 \text{ marks})$