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FOURTH SEMESTER (CBCSS–UG) DEGREE EXAMINATION, APRIL 2022

Mathematics

MTS 4B 04—LINEAR ALGEBRA

(2019 Admission onwards)

Time : Two Hours and a Half Maximum : 80 Marks

Section A (Short Answer Type Questions)

Answer at least **ten** *questions.*
Each question carries 3 *marks.*
All questions can be attended.
Overall Ceiling 30.

1. Show that the linear system of equations $\begin{matrix} 4x-2y=1 \\ 16x-8y=4 \end{matrix}$ has infinitely many solutions.

2. Write any two facts about row echelon forms and reduced row echelon forms.

3. Express the linear system
$$\begin{aligned} 4x_1-3x_3+x_4&=1 \\ 5x_1+x_2-8x_4&=3 \\ 2x_1-5x_2+9x_3-x_4&=0 \\ 3x_2-x_3+7x_4&=2 \end{aligned}$$
in the form AX = B.

4. Let V = R^2 and define addition and scalar multiplication as follows. For $\bar{u}=(u_1,u_2),\bar{v}=(v_1,v_2)$, $\bar{u}+\bar{v}=(u_1+v_1,u_2+v_2)$ and for a real number $k, k\bar{u}=(ku_1,0)$. For $\bar{u}=(1,1)$ and $\bar{v}=(-3,5)$ find $\bar{u}+\bar{v}$ and for k = 5, find $k\bar{u}$. Also show that one axiom for vector space is not satisfied.

5. Define basis for a vector space.

6. How will you relate the dimension of a finite dimensional vector space to the dimension of its subspace. Give two facts.

7. Give a solution to the change of basis problem.

8. When you can say that a system of linear equation Ax = b is consistent. What is meant by a particular solution of the consistent system Ax = b.

9. Find the rank of a 5 × 7 matrix A for which Ax = 0 has a two-dimensional solution space.

Turn over

10. If $T_A : R^n \to R^m$ is a matrix transformation. Then define its kernel ker (T_A) and Range of (T_A). What is ker (T_A) in terms of null-space of A.

11. Discuss the geometric effect on the unit square of multiplication by a diagonal matrix $A = \begin{bmatrix} k_1 & 0 \\ 0 & k_2 \end{bmatrix}$.

12. Confirm by multiplication that x is an eigen vector of A and find the corresponding eigen value, if $A = \begin{bmatrix} 5 & -1 \\ 1 & 3 \end{bmatrix}$ and $x = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

13. Let R^2 have the weighted Euclidean inner product $< u,v > = 2u_1v_1 + 3u_2v_2$. For $u = (1,1), v = (3,2)$, compute $d(u,v)$.

14. If u and v are orthogonal vectors in a real inner product space, then show that $\| u+v \|^2 = \| u \|^2 + \| v \|^2$.

15. State four properties of orthogonal matrices.

(10 × 3 = 30 marks)

Section B (Paragraph/ Problem Type Questions)

Answer at least **five** *questions.*
Each question carries 6 *marks.*
All questions can be attended.
Overall Ceiling 30.

16. Suppose that the augmented matrix for a linear system has been reduced to the row echelon form as $\begin{bmatrix} 1 & 0 & 8 & -5 & 6 \\ 0 & 1 & 4 & -9 & 3 \\ 0 & 0 & 1 & 1 & 2 \end{bmatrix}$ solve the system.

17. If A is an invertible matrix, then show that A^T is also invertible and $\left(A^T\right)^{-1} = \left(A^{-1}\right)^T$.

18. Let V be a vector space and $\bar{u}$, a vector in V and k a scalar. Then show that (i) $O\bar{u} = 0$; (ii) $(-1)\bar{u} = -\bar{u}$.

19. If $S = [v_1, v_2, \ldots, v_n]$ is a basis for a vector space V, then show that every vector v in V can be expressed in form $v = c_1v_1 + c_2v_2 + \ldots + c_nv_n$ in exactly one way. What are the co-ordinates of V relative to the basis S ?

20. Consider the basis $B = [u_1, u_2]$ and $B' = [u_1', u_2']$ for R^2, where $u_1 = (2,2)$ $u_2 = (4,-1)$ $u_1' = (1,3)$ $u_2' = (-1,-1)$.

    (a) Find the transition matrix from B' to B.

    (b) Find the transition matrix from B to B'.

21. If A is a matrix with n columns, then define rank A, nullity of A and establish a relationship between them.

22. Define eigen space corresponding to an eigen value λ of a square matrix A. Also find eigen value and bases for the eigen space of the matrix $A = \begin{bmatrix} 1 & -2 \\ 0 & 1 \end{bmatrix}$.

23. Use the Gram–Schmidt process for an orthonormal basis corresponding to the basis vectors $u_1 = (1,1,1)$, $u_2 = (0,1,1)$ and $u_3 = (0,0,1)$.

(5 × 6 = 30 marks)

Section C (Essay Type Questions)

Answer any **two** *questions.*
Each question carries 10 *marks.*

24. Show that the following statements are equivalent for an $n \times n$ matrix A :

    (a) A is invertible.

    (b) $Ax = 0$ has only the trivial solution.

    (c) The reduced row echelon form of A is I_n.

    (d) A is expressible as a product of elementary matrices.

25. (a) Define Wronskian of the functions $f_1 = f_1(x), f_2 = f_2(x) \ldots f_n = f_n(x)$ which are $n - 1$ times differentiable in $(-\infty, \infty)$. Use this to show that $f_1 = x$ and $f_2 = \sin x$ are linearly independent vectors in $c^{\infty}(-\infty, \infty)$.

    (b) Show that the vectors $v_1 = (1,2,1), v_2 = (2,9,0)$ and $v_3(3,3,4)$ form a basis for R^3.

Turn over

26. (a) If A is the matrix $\begin{bmatrix} 1 & -2 & 0 & 0 & 3 \\ 2 & -5 & -3 & -2 & 6 \\ 0 & 5 & 15 & 10 & 0 \\ 2 & 6 & 18 & 8 & 6 \end{bmatrix}$, then find a basis for the row space consisting on entirely row vectors from A.

(b) Find the standard matrix for the operator $T : R^3 \to R^3$ that first rotates a vector counter clockwise about z-axis through an angle θ, reflects the resulting vector about yz plane and then projects that vector orthogonally onto the xy plane.

27. (a) On P_2, polynomial in $[-1,1]$, define innerproduct as $< p,q > = \int_{-1}^{1} p(x)q(x)\,dx$. Find $\| p \|, \| q \|$ and $< p,q >$ for $p = x$ and $q = x^2$.

(b) If A is an $n \times n$ matrix with real entries, show that A is orthogonally diagonalizable if and only if A has an orthonormal set of n eigenvectors.

(2 × 10 = 20 marks)