D 10230 **(Pages : 3)** **Name..................................**

Reg. No...............................

FIFTH SEMESTER B.A./B.Sc. DEGREE EXAMINATION, NOVEMBER 2021

(CUCBCSS-UG)

Mathematics

MAT 5B 05—VECTOR CALCULUS

Time : Three Hours Maximum : 120 Marks

Part A

Answer **all** *questions.*
Each question carries 1 *mark.*

1. Find the domain and range of $z = \sqrt{25 - x^2 - y^2}$.
2. Evaluate $\lim\limits_{(x,y)\to(1,-1)} \dfrac{1+x-y}{2-x+y}$.
3. Define gradient of a scalar function.
4. Compute the divergence of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$.
5. Define solenoidal vector.
6. What do you mean by directional derivative.
7. Write the component test for the differential M(x, y, z) dx + N(x, y, z) dy + P(x, y, z) dz to be exact.
8. Find du if $u = \arcsin \dfrac{x}{y}$.
9. Fill in the blanks : If $\vec{f}$ and $\vec{g}$ are irrotational vector point functions, then $\nabla.\left(\vec{f} \times \vec{g}\right) = \ldots.$
10. State the normal form of Green's theorem in the plane.
11. Fill in the blanks : If $\vec{a}$ is a constant vector and $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$, then of $\nabla\left(\vec{r}.\vec{a}\right) = \ldots.$
12. State Stoke's theorem.

(12 × 1 = 12 marks)

Part B

Answer any **ten** *questions.*
Each question carries 4 *marks.*

13. Evaluate $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2 - xy}{\sqrt{x} - \sqrt{y}}$.

Turn over

14. Find the vector normal to the surface $\phi(x, y, z) = xyz$ at $(1, -1, 1)$.

15. Find $\frac{\partial z}{\partial x}$ and $\frac{\partial z}{\partial y}$ at (π, π, π) from $\sin(x + y) + \sin(y + z) + \sin(x + z) = 0$.

16. Prove that $\nabla\left(r^n\right) = nr^{n-2}\vec{r}$.

17. Compute the average value of the function $f(x, y, z) = xyz$ over the boundary of the cube $0 \le x \le 2, 0 \le y \le 2, 0 \le z \le 2$.

18. Evaluate $\int_1^2 \int_3^4 \frac{1}{(x + y)^2}\, dx\, dy$.

19. Leaniarize the function $f(x, y, z) = xy + yz + zx$ at (1, 1, 1).

20. Find the directional derivative of $f(x, y, z) = xy$ at (1, 2).

21. Evaluate $\iint_R (xy)\, dx\, dy$ where R is the positive quadrant of the circle of radius a centred at the origin.

22. Find the flow of $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ along the portion of the circular helix $x = \cos t, y = \sin t, z = t; 0 \le t\, \pi/2$.

23. Test whether $\overline{f} = (yz)\,\vec{i} + (xz)\,\vec{j} + (xy)\,\overline{k}$ is conservative or not.

24. Prove that $div\left(curl\overline{f} = 0\right)$.

25. Verify whether the differential $\left(e^x \cos y + yz\right) dx + \left(xz - e^x \sin y\right) dy\, (xy + z)\, dz$ is exact or not.

26. If S is a closed surface enclosing a volume V then prove that $\iint_S \overline{culf} \cdot \hat{n} dS = 0$.

(10 × 4 = 40 marks)

Part C

Answer any **six** *question.*
Each question carries 7 marks.

27. Using double integrals prove that $\int_0^\infty e^{-\left(x^2\right)} dx = \sqrt{\pi}/2$.

28. Evaluate the line integral $\int_C ydx + xdy$ where C is the boundary of the square x = 0, x = 1, y = 0 and y = 1.

29. Find the work done by the force field $\vec{f} = 3xy\vec{i} - 58\vec{j} + 10x\vec{k}$ along the space curve $C: \vec{r} = (t^2+1)\vec{i} + 2t^2\vec{j} + t^3\vec{k}$ where $0 \le t \le 2\pi$.

30. Find angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at (2, – 1, 2).

31. Evaluate the volume bounded by $y = x^2, x = y^2$ and the planes $z = 0$ and $z = 3$.

32. Evaluate the area enclosed by the region cut from the plane $x + 2y + 2z = 5$ by the cylinder whose walls are $x = y^2$ and $x = 2 - y^2$.

33. Find the Local extreme values of $f(x, y) = x^2 + y^2 + xy + 3x - 3y + 4$.

34. Evaluate the line integral $\int_C \vec{f} \cdot \vec{dr}$ where C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0), (1, 1, 0).

35. Show that $\vec{f} = y \sin z\vec{i} + x \sin z\vec{j} + xy \cos z\vec{k}$ is conservative and find its scalar potential.

(6 × 7 = 42 marks)

Part D

Answer any **two** *question.*
Each question carries 13 *marks.*

36. (a) State Gauss divergence theorem and use it to evaluate the outward flux of $\vec{f} = xy\vec{i} + yz\vec{j} + xz\vec{k}$ through the surface of the cube cut from the first Octant by the planes $x = y = z = 1$.

(b) Evaluate $\int_{(1,0,0)}^{(0,1,0)} \sin y \cos x dx + \cos y \sin x dy + dz$.

37. Verify Stoke's Theorem for $\vec{f} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$ over the rectangular region bounded by $x = 0, x = a, y = 0, y = a$.

38. Verify the Tangential form of Green's theorem in the plane for the vector Field $\vec{f} = (x - y)\vec{i} + x\vec{j}$ over the region bounded by the unit circle $x^2 + y^2 = 1$.

(2 × 13 = 26 marks)