C 21296 **(Pages : 4)** Name..................................

Reg. No...............................

FOURTH SEMESTER (CUCBCSS—UG) DEGREE EXAMINATION
APRIL 2022

Mathematics

MAT 4B 04—THEORY OF EQUATION, MATRICES AND VECTOR CALCULUS

(2014—2018 Admissions)

Time : Three Hours Maximum : 80 Marks

Part A (Objective Type)

Answer **all** *twelve questions.*

Each question carries 1 mark.

1. State the fundamental theorem of theory of equations.
2. If α, β, γ are the roots of the equation $ax^3 + bx^2 + cx + d = 0$, write the equation whose roots are $-\alpha, -\beta, -\gamma$.
3. Find the number of real roots of $x^4 - 1 = 0$.
4. Write the standard form of a cubic equation.
5. Find the rank of $\begin{bmatrix} 1 & 2 & 1 \\ 1 & 1 & 1 \end{bmatrix}$.
6. If A and B are non-singular square matrices of order 5, find the rank of AB.
7. Find the number of solutions of the equation $x + 2y = 3$.
8. If $A = \begin{bmatrix} 1 & 0 & 0 \\ 2 & 4 & 2 \\ a & 6 & b \end{bmatrix}$ and the system of homogeneous linear equations AX = 0 has a non-zero solution, find the value of b.
9. Find the characteristic roots of $\begin{bmatrix} 2 & 0 \\ 0 & 5 \end{bmatrix}$.
10. Find the parametric equations of the line through the point (3, – 4, – 1) parallel to the vector $\mathbf{i} + \mathbf{j} + \mathbf{k}$.

Turn over

11. Find the angle between the planes $x + y = 1$, $2x + y - 2z = 2$.

12. Find the unit tangent vector to the curve $\mathbf{r}(t) = (\cos t)\,\mathbf{i} + (\sin t)\,\mathbf{j}$.

(12 × 1 = 12 marks)

Part B (Short Answer Type)

Answer any **nine** *questions.*
Each question carries 2 *marks.*

13. If α, β, γ are the roots of the equation $2x^3 + x^2 - 2x - 1 = 0$, find the value of $\alpha + \beta + \gamma$.

14. If α, β, γ are the roots of the equation $2x^3 + 3x^2 - x - 1 = 0$, find the equation whose roots are $\frac{1}{2\alpha}, \frac{1}{2\beta}, \frac{1}{2\gamma}$.

15. Show that the equation $x^4 + 4x^2 + 5x - 6 = 0$ has exactly one positive root.

16. Show that the rank of a matrix, every element of which is unity is 1.

17. Find the normal form of $\begin{bmatrix} 1 & 2 & -3 \\ 2 & 5 & -4 \end{bmatrix}$.

18. Find the values of λ so that the system of equations $\lambda x + y = 0$, $x + \lambda y = 0$ has zero solution only.

19. Prove that the characteristic roots of triangular matrix are the same as its diagonal elements.

20. Show that if λ is a characteristic root of a matrix A, then $\lambda + k$ is a characteristic root of the matrix $A + kI$.

21. Find the spherical co-ordinate equation for the sphere $x^2 + y^2 + (z - 1)^2 = 1$.

22. If $\mathbf{r}(t) = (3\cos t)\,\mathbf{i} + (3\sin t)\,\mathbf{j} + t^2\mathbf{k}$ is the position vector of a particle in space at time t, at what times, if any, are the body's velocity and acceleration orthogonal ?

23. If u is a differentiable vector function of t of constant magnitude, prove that $\mathbf{u} \cdot \frac{du}{dt} = 0$.

24. Show that the curvature of a straight line is zero.

(9 × 2 = 18 marks)

Part C (Short Essay Type)

Answer any **six** *questions.*
Each question carries 5 *marks.*

25. Solve $4x^3 - 24x^2 + 23x + 18 = 0$, given that the roots are in A.P.

26. If α, β, γ are the roots of the equation $x^3 + 3x^2 + 6x + 1 = 0$, find the value of $(\alpha^2 + 1)(\beta^2 + 1)(\gamma^2 + 1)$.

27. Obtain the real root of the equation $x^3 - 15x = 126$ by Cardan's method.

28. Reducing to the normal form, find the rank of $\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$.

29. If $A = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & -1 \end{bmatrix}$, find non-singular matrices P and Q such that PAQ is in normal form.

30. Test for consistency and solve the system of equations :

$$5x + 3y + 7z - 4 = 0$$

$$3x + 26y + 2z - 9 = 0$$

$$7x + 2y + 10z - 5 = 0.$$

31. If A is a non-singular matrix, prove that the eigenvalues of A^{-1} are the reciprocals of the eigenvalues of A.

32. Find the distance from the point (1, 1, 5) to the line $x = 1 + t, y = 3 - t, z = 2t$.

33. Obtain the curvature of a circle of radius a.

($6 \times 5 = 30$ marks)

Turn over

Part D (Essay Type)

Answer any **two** *questions.*
Each question carries 10 *marks.*

34. Solve $6x^5 + 11x^4 - 33x^3 - 33x^2 + 11x + 6 = 0$.

35. Find the characteristic roots and the corresponding characteristic vectors for the matrix

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}.$$

36. Find the binormal vector and torsion for the space curve $\mathbf{r}(t) = (3 \sin t)\,\mathbf{i} + (3 \cos t)\,\mathbf{j} + 4t\mathbf{k}$.

(2 × 10 = 20 marks)